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# SF-6463

## B. E. - II (Sem. IV) (Comp./I.T.) Examination

**May / June – 2011**

## Mathematics : Paper - III

Time : Hours] [Total Marks :

**Instructions :**

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
**Fillup strictly the details of ☛ signs on your answer book.**

Name of the Examination :

☛ **B. E. - 2 (Sem. 4) (Comp./I.T.)**

Name of the Subject :

☛ **Mathematics - 3**

☛ Subject Code No. : **6 4 6 3** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt **all** questions.

(3) Figures to right indicate the maximum marks of the question.

**1** (a) Do as directed :

(1) Evaluate $\int_{1}^{4}\int_{\sqrt{y}}^{2}\left(x^2+y^2\right)dA$ by changing the order of integration. **3**

(2) Prove that $\nabla f(r)=\frac{f'(r)}{r}\bar{r}$, where, $\bar{r}=xi+yj+zk$. **3**

(3) Evaluate $\int_{0}^{a}\int_{0}^{\sqrt{a^2-x^2}}\int_{0}^{a^2-x^2-y^2} x^2\,dz\,dy\,dx$. **4**

(b) Attempt any **three** : **12**

(1) Evaluate the integral $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}}\left(x^2+y^2\right)dx\,dy$.

(2) Find the area of the region enclosed by the cardiods $r=a(1+\cos\theta)$ and $r=a(1-\cos\theta)$.

(3) Find the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$.

(4) Find the surface area of the portion of the paraboloid $z=x^2+y^2$ below the plane $z=1$.

**2** (a) Attempt any **two** : **6**

(1) Find the directional derivative of $\phi=\bar{F}^2$ where $\bar{F}=xy^2i+zy^2j+xz^2k$. at the point $(2, 0, 3)$ in the direction of the outward drawn normal vector to the sphere. $x^2+y^2+z^2=14$ at the point $(3, 2, 1)$.

(2) A fluid motion is given by

$\bar{V}=(y\sin z-\sin x)i+(x\sin z+2yz)\,j+\left(xy\cos z+y^2\right)k$

Is the motion irrotational ? If so, find the velocity potential.

(3) Prove that $r^n\,\bar{r}$ is irrotational and is solenoidal when $n=-3$.

(b) Attempt any **two** : **8**

(1) Using Green's theorem evaluate

$\oint\left[\left(xy-x^2\right)dx+x^2y\,dy\right]$ along the closed cutve $C$

formed by $y=0,\ x=1$ and $y=x$.

(2) Use Gauss' divergence theorem to evaluate

$\iint_S \bar{F}\cdot\hat{n}\,ds$ where $\bar{F}=4xi-2y^2j+z^2k$ and $S$ is the surface bounding the region $x^2+y^2=4;\ z=0$ and $z=3$.

(3) Evaluate $\oint_C \bar{F}\cdot d\bar{r}$ by Stoke's theorem, where $\bar{F}=y^2i+x^2j-(x+z)k$ and $C$ is the boundry of the triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0).

**3** (a) Explain the half range Fourier series. **4**

(b) Attempt any **two** : **10**

(1) Obtain Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$.

(2) Prove that $\frac{1}{2} - x = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{2n\pi x}{l}$, $0 < x < l$.

(3) If $f(x) = -1 + x, \quad -\pi < x < 0$

$= 1 + x, \quad 0 < x < \pi$

Find the Fourier series of $f(x)$.

4 (a) Attempt the following : **10**

(1) State one dimensional Wave equation and express its solution in terms of Fourier coefficients.

(2) Show that Beta function is symmetric.

(3) Derive the Cauchy-Riemann equations in polar coordinates.

(4) State the Fourier series $f(x)$ in the interval $\alpha < x < \alpha + 2l$ with its Fourier co-efficients.

(5) Define Error function and complementary Error function.

(b) Attempt any **two** : **6**

(1) Prove that $\int_0^{\infty}\frac{x^a}{a^x}dx = \frac{\Gamma(a+1)}{(\log a)^{a+1}}$

(2) Evaluate $\int_0^{\infty}\frac{x^4}{(1+x^2)^4}dx$

(3) Prove that $\int_0^{\infty} e^{-(x+a)^2}\,dx = \frac{\sqrt{\pi}}{2}\left[1 - erf(a)\right]$

(c) Solve any **two** : **6**

(1) $pz - qz = z^2 + (x+y)^2$

(2) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = xyz$

(3) $(x^2 - y^2 - z^2)\,p + 2xyq = 2xz$

5 Attempt any **two** : **14**

(1) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions :

$u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\frac{n\pi x}{l}$.

(2) Determine the solution of the one dimensional Heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, where the boundary conditions are $u(0, t) = 0$, $u(l, t) = 0\ (t > 0)$ and the initial condition $u(x, 0) = x$, $l$ being the length of the bar.

(3) A string is stretched and fastened to two points 1 apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance $x$ from one end at time $t$ is given by $y(x, t) = a \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}$.

**6** (a) Attempt any **two** : **8**

(1) Determine the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$.

(2) Determine the analytic function $w = u + iv$, if $v = \log(x^2 + y^2) + x - 2y$.

(3) If $f(z)$ is a regular function of $z$, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$.

(b) Attempt any **two** : **6**

(1) Evaluate $\int_0^{2+i} (\bar{Z})^2 \, dz$, along the real axis to 2 and then vertically to $2 + i$.

(2) Evaluate $\int_C \frac{3z^2 + z}{z^2 - 1} \, dz$, where $C$ is the circle $|z - 1| = 1$.

(3) Evaluate $\int_0^{1+i} (x^2 - iy) \, dz$, along the path $y = x^2$.